

Real Options: Black-Scholes?

What are real options?

Real option valuation is a pricing technique that takes account of the uncertainty of cash flows prediction and the management's ability to react to a changed environment. In particular, the management is able to abandon a project or to divest if the project is not profitable anymore, or to invest once the project turns profitable. Other decisions that depend on the profitability of a business opportunity are options to expand, options to switch, options to wait, or options to shut down (as a reference consult e.g. (1)). At the very root of these options is always an investment that is due or that can be saved. These options (called "real options", because they are reality) have similar payoff profiles like financial options.

What is Black Scholes?

Fisher Black and Myron Scholes developed in their seminal paper (2) a formula to price financial options like calls or puts. One crucial hypothesis in their derivation is the possibility to replicate the option with the underlying and a bond. This means that the holder of the option holds at the same time a portfolio that is designed to eliminate the risk stemming from the option (called "hedging"). Although Black and Scholes start with hypothesis that the underlying stock can have an arbitrary rate of return μ , this μ does not appear anymore in the formula but gets replaced with the risk free interest rate r .

$$V(S, K, T, r, \sigma) = SN \left(\frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \right) - Ke^{-rT} N \left(\frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \right)$$

Black-Scholes formula for a call option, S being the underlying stock, K the strike, T the maturity, r the risk free interest rate, and σ the volatility.

Can we use Black-Scholes for real options?

No. In most cases of real options the underlying is future revenues. These are usually not tradable. Often the holder of the real option is even the only company having that option (e.g. only the holder of the patent may commercialise the product). Therefore, the replication hypothesis of the Black-Scholes formula does not hold. In some industries like gold mining or oil drilling it is possible to build a replicating portfolio since gold and petrol have very liquid market. However, the Black-Scholes formula does not only require that you *can* hedge your option, it requires much more that you *do* hedge the option. It is not known that any company holds such a replicating portfolio.

As a consequence we cannot use the Black-Scholes formula. While the price of a financial option is the result of a trading strategy, the value of a real option is simply the discounted expected payoff of the business case. The above formula would become:

$$V(S, K, T, r, \mu, \sigma) = S e^{(\mu-r)T} N \left(\frac{\ln\left(\frac{S}{K}\right) + \left(\mu + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \right) - Ke^{-rT} N \left(\frac{\ln\left(\frac{S}{K}\right) + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \right)$$

Value of a call-like real option. μ being the growth rate of the underlying S , r the discount rate.

Anyway, most business cases are more complex than just a simple call option analogy. As a reference for drug development projects and another description of the theme "Black-Scholes and real options" consult (3).

Literature

1. Copeland, T., Real Options. A Practitioners Guide. bh
2. Black, F. and M. Scholes, The pricing of options and corporate liabilities. Journal of Political Economy, 1973 (81), pp.637-654
3. Villiger R. and B. Bogdan, Pitfalls of Valuation. Journal of Commercial Biotechnology, 2006 (12) 3, April.