

Black's Discounting Rule

The discount rate – or the cost of capital – is the most influential parameter in a valuation and unfortunately only observable to a very limited degree. This makes valuations somewhat random, as long as there is no clear concept underlying the chosen cost of capital. Ideally we discount with an observable rate, e.g. the risk-free rate. In 1988 Fischer Black, the first half in Black-Scholes, came up with a simple rule that allows discounting at the risk-free rate.

The theory

Let us assume that the net cash flows (NCF) of the company for each time period can be forecasted in the following way:

$$NCF = \alpha + \beta r_M + \varepsilon$$

Where α is the intercept, r_M the return of a benchmark security (can be more or less any security), β is the sensitivity to that benchmark security, and ε is the idiosyncratic (diversifiable) risk. Rearranging we can split it into a known and an unknown quantity:

$$NCF = \alpha + \beta r_f + \beta(r_M - r_f) + \varepsilon,$$

where r_f is the risk-free rate. In this equation we have an equivalent to the risk-premium from the capital asset pricing model (CAPM): $r_M - r_f$. It is easy to see that when we suppose $r_M = r_f$ then the conditional expectation of NCF becomes:

$$E(NCF | r_M = r_f) = \alpha + \beta r_f$$

Since this expression is a known quantity, i.e. without uncertainty, we can discount it at the risk-free rate. The value of a project or company therefore becomes the sum of these conditional net

cash flows discounted at the risk-free rate.

Now we only have to know what these conditional net cash flows actually mean. One approach to determine these conditional cash flows, which is in general a pretty uncommon metric, is to assume the following: The benchmark security underperforms the risk-free rate only with a certain probability ϕ . We therefore take as our cash flows estimate the ϕ -quantile of the distribution of our net cash flows. The following figure illustrates the concept.

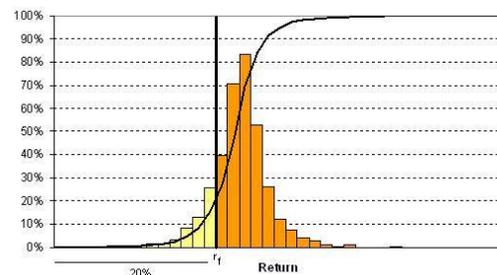


Figure 1: Benchmark security return distribution

It suffices now to determine the quantile for which $r_M = r_f$ (in figure 1 this quantile would be the 20% percentile) and use the same quantile of the NCF-distribution as cash flow estimates. These cash flows can then be discounted at the risk free rate.

Loderer et al. (1992) have observed that these quantiles are pretty constant over various markets and securities. However, it is important to mention that the time horizon matters quite a bit. While for 1-year returns the critical quantile is about 34%, it drops to only 20% for a 5-year return. For a life science investment the time horizon is typically rather long, even 10 to 20 years. This means that we would have to use a percentile as low as

10% down to 4% for our conditional cash flows events. Does this still make sense?

Table 1: Quantiles depending on time horizon

Years	r_M	$\sigma(r_M)$	r_f	Quantile
1	11%	15%	5%	34%
5	55%	34%	25%	20%
10	110%	47%	50%	10%
15	165%	58%	75%	6%
20	220%	67%	100%	4%
n	$n \cdot r_M$	$\text{Sqrt}(n) \cdot \sigma(r_M)$	$n \cdot r_f$	1

In theory yes, as usually we would discount all along with a high cost of capital, up to 15% higher than the risk free rate, this punishment gets proportionally higher and higher the longer the time horizon. Therefore it is natural that we have to use a lower quantile for longer time horizons. Table 1 explains how to calculate the quantile². Figure 2 displays how the percentile decreases the more the net cash flow lies in the future.

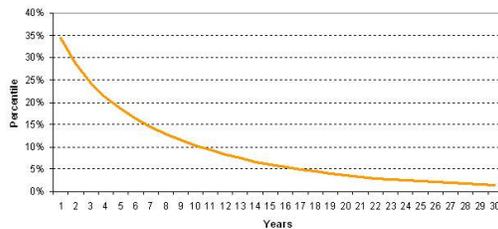


Figure 2: Percentile depending on time of net cash flow.

Extension to life sciences

As soon as it comes to probability distributions matters complicate in life sci-

¹ In Excel: `normdist(rf, rM, σ(rM), TRUE)`

² We assume normal distribution of returns. This is relatively standard in financial theory, especially in CAPM and modern portfolio theory. We also used continuous compounding instead of discrete compounding.

ences as we have two probability concepts that apply; the success rates and the uncertainty of the cash flows. The success rates refer to whether the project can be continued or not, the uncertainty of the cash flows refers to the actual size of the cash flows in case they actually occur. Now, the 20% percentile for most net cash flows is zero, as it will reach that stage with less than 20% probability. Well, it doesn't work like that. In general we can assume that technical (success rates) and economic uncertainty are independent of each other. All net cash flows out of the distribution are equally likely to be a victim of attrition. Consequently, it suffices to assume percentile of the risk-adjusted net cash flows for the valuation; or the risk-adjusted percentile of the net cash flow.

In general most cash flows can be estimated to a good degree except sales. But sales can have quite diverse scenarios. It is therefore a reasonable shortcut to assume all cash flows as fix except sales. And for the sales we take the indicated percentile. For our example we can even use percentiles for the R&D costs, as DiMasi's paper (2003) indicates the probability distribution of the costs as well.

Application

In reality it is, of course, quite difficult to estimate the net cash flow corresponding to a certain percentile. One might have an idea about sales in a certain disease area in general, but if you are able to specify the project already a bit more in detail, e.g. a niche product, it

becomes more difficult to estimate the distribution of the sales.

Grabowski and Vernon (2000) have presented to some extent the distribution of sales in 1990 USD of drug projects (cf figure 3).

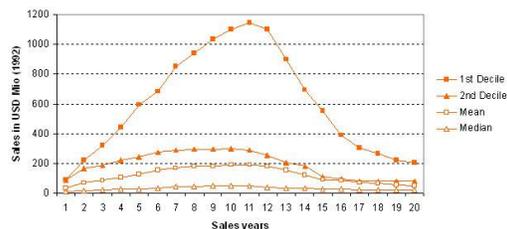


Figure 3: Sales distribution after Grabowski and Vernon (2000).

One could extract the probability distribution of sales from this kind of data, also one has to be careful about the statistics; what has been counted as a drug, is that comparable to what we want to value, etc. In figure 3, for instance, median sales seem very low.

We value a preclinical oncology project with assumptions as laid out in table 2.

Table 2: Assumptions for an oncology R&D project.

	Preclinical	P1	P2	P3	Review
Duration	1.5	2	2	3	1.5
Success Rate	77%	77%	44%	58%	90%
Cost	-2	-5	-11	-40	-3
St. dev	1	4	8	20	2
Distribution	normal	normal	normal	normal	normal

The sales curve has a similar shape like in figure 3. We have assumed a lognormal distribution for cancer peak sales, with median peak sales of USD 618 Mio.

Using these assumptions and a continuously compounded risk free rate of 5% we receive a value of USD 8.3 Mio for a preclinical oncology project. Using the same assumptions and mean oncology sales, this corresponds to a continuously compounded discount rate of 23%

(25.9% discretely compounded). The calculation sheet is downloadable on www.avance.ch/knowledge_down.html.

Discussion

A sensitivity analysis shows that the value is reasonably sensitive to changes in the project's parameters such as timelines, costs, or success rates. But the value reacts heavily to changes in either the risk-free rate and benchmark security or the probability distribution. The reason for this is without any doubt the very long timeline of a drug development project.

With quantitative finance one can get the necessary data with respect to quantiles and risk-free rate from traded securities such options on the benchmark security and government bonds. However, in order to get the right probability distribution this requires very sophisticated methods. The probability distribution of the sales (or net cash flows in general) is much more difficult to get hold of. The main problem is that given the long time-horizon, it is mainly the tail of the distribution that matters (in our case we need the 10% down to 2% percentiles). Unfortunately, probability distributions are extremely sensitive in their tails and we do not have a reasonable sample of good data points (i.e. sales data) that would make us comfortable with this issue.

Conclusion

Black's discounting rule is an interesting approach, although it relies on some daring assumptions; but every model does. However, the long timelines of drug development projects and the

problem of the tail of probability distributions make this valuation method unsuitable for life science valuation.

References

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